Preface

In this Project we discuss short, but comprehensive introduction to Methods of computation Pi some of learning rules. Chapter 2 consists of a brief discussion on Pi and history of Pi. Chapter 3 consists of methods of computation pi. In which chapter we discuss how to calculate value of pi using so many methods and also design applet for help to be easily understand how do work and how do calculate value of Pi.

The work of Applet is basically divided into two parts: the first part deals with diagram using Java, HTML and LaTeX and the second part deals with theorey of Applet how to work Applet for calculating value of Pi.
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Chapter 1

History Of $\pi$

1.1 Introduction of $\pi$

Four thousand years ago people discovered that the ratio of the circumference of a circle to its diameter was about 3. In nature people saw circles, great and small, and they realized that this ratio was an important tool.

This tool was used by the Babylonians and the Egyptians. Reference is made to the concept of $\pi$ in the Bible. The Chinese found a value of $\pi$ that stood for one thousand years. One man felt the accomplishment of taking $\pi$ to 35 places was the most important achievement of his life, so much so, that he had it inscribed on his epitaph. With the help of $\pi$ computers has been taken to over 6 billion places. People have been fascinated by $\pi$, an irrational number, throughout history.

little known verse of the Bible reads

"And he made a molten sea, ten cubits from the one brim to the other: it was round all about, and his height was five cubits: and a line of thirty cubits did compass it about." (I Kings 7, 23)
The same verse can be found in II Chronicles 4, 2. It occurs in a list of specifications for the great temple of Solomon, built around 950 BC and its interest here is that it gives $p = 3$. Not a very accurate value of course and not even very accurate in its day, for the Egyptian and Mesopotamian values of $25/8 = 3.125$ and $\sqrt{10} = 3.162$ have been traced to much earlier dates: though in defence of Solomon’s craftsmen it should be noted that the item being described seems to have been a very large brass casting, where a high degree of geometrical precision is neither possible nor necessary. There are some interpretations of this which lead to a much better value.

The fact that the ratio of the circumference to the diameter of a circle is constant has been known for so long that it is quite untraceable. The earliest values of $p$ including the ‘Biblical’ value of 3, were almost certainly found by measurement. In the Egyptian Rhind Papyrus, which is dated about 1650 BC, there is good evidence for $4(8/9)^2 = 3.16$ as a value for $p$.

The first theoretical calculation seems to have been carried out by Archimedes of Syracuse (287-212 BC). He obtained the approximation . . .

$$223/71 < p < 22/7.$$  

notice that very considerable sophistication involved in the use of inequalities here. Archimedes knew, what so many people to this day do not, that $p$ does not equal $22/7$, and made no claim to have discovered the exact value. If we take his best estimate as the average of his two bounds we obtain 3.1418, an error of about 0.0002.$p$

The European Renaissance brought about in due course a whole new mathematical world. Among the first effects of this reawakening was the emergence of mathematical formulae for $p$. One of the earliest was that of Wallis (1616-1703)

$$2/p = (1.3.3.5.5.7.\ldots)/(2.2.4.4.6.6.\ldots)$$

and one of the best-known is . . .

$$p/4 = 1-1/3+1/5-1/7+\ldots$$
This formula is sometimes attributed to Leibniz (1646-1716) but is seems to have been first discovered by James Gregory (1638-1675).

These are both dramatic and astonishing formulae, for the expressions on the right are completely arithmetical in character, while $\pi$ arises in the first instance from geometry. They show the surprising results that infinite processes can achieve and point the way to the wonderful richness of modern mathematics.

From the point of view of the calculation of $\pi$, however, neither is of any use at all. In Gregory’s series, for example, to get 4 decimal places correct we require the error to be less than $0.00005 = 1/20000$, and so we need about 10000 terms of the series. However, Gregory also showed the more general result

\[
\tan^{-1} x = x - x^3/3 + x^5/5 - \ldots (-1 \leq x \geq 1)
\]

from which the first series results if we put $x = 1$. So using the fact that $\tan^{-1}(1/\sqrt{3}) = \pi/6$ we get

\[
\pi/6 = (1/\sqrt{3})(1-1/(3.3)+1/(5.3.3)-1/(7.3.3.3)+\ldots)
\]

which converges much more quickly. The 10th term is $1/19 \times 39\sqrt{3}$, which is less than $0.00005$, and so we have at least 4 places correct after just 9 terms.

An even better idea is to take the formula

\[
\pi/4 = \tan^{-1}(1/2) + \tan^{-1}(1/3)
\]

and then calculate the two series obtained by putting first $1/2$ and the $1/3$ into (3). Clearly we shall get very rapid convergence indeed if we can find a formula something like

\[
\pi/4 = \tan^{-1}(1/a) + \tan^{-1}(1/b)
\]
with $a$ and $b$ large. In 1706 Machin found such a formula:

$$p/4 = 4\tan^{-1}(1/5) - \tan^{-1}(1/239)$$

Actually this is not at all hard to prove, if you know how to prove (4) then there is no real extra difficulty about (5), except that the arithmetic is worse. Thinking it up in the first place is, of course, quite another matter.

We conclude with one further statistical curiosity about the calculation of $p$, namely Buffon’s needle experiment. If we have a uniform grid of parallel lines, unit distance apart and if we drop a needle of length $k < 1$ on the grid, the probability that the needle falls across a line is $2k/p$. Various people have tried to calculate $p$ by throwing needles. The most remarkable result was that of Lazzerini (1901), who made 34080 tosses and got

$$p = \frac{355}{113} = 3.1415929$$

which, incidentally, is the value found by Zu Chongzhi. This outcome is suspiciously good, and the game is given away by the strange number 34080 of tosses. Kendall and Moran comment that a good value can be obtained by stopping the experiment at an optimal moment. If you set in advance how many throws there are to be then this is a very inaccurate way of computing $p$. Kendall and Moran comment that you would do better to cut out a large circle of wood and use a tape measure to find its circumference and diameter.

Still on the theme of phoney experiments, Gridgeman, in a paper which pours scorn on Lazzerini and others, created some amusement by using a needle of carefully chosen length $k = 0.7857$, throwing it twice, and hitting a line once. His estimate for $p$ was thus given by.

$$2 \times 0.7857 / p = 1/2$$

from which he got the highly creditable value of $p = 3.1428$. He was not being serious!
1.2 What is \( \pi \)

(1) Pi is the sixteenth letter of the Greek alphabet, but the lower case symbol is used to represent a special mathematical constant.

(2) the ratio of the circumference to the diameter of a circle

(3) the 16th letter of the Greek alphabet

(4) Pi is the number of times the diameter divides into the circumference of a circle. It is approximately 3.14159 times. (3.14)

(5) Each NERSC repository has a single Principal Investigator, or PI for short. The PI is the scientific head of the project supported by an allocation of NERSC resources. Although the PI may delegate some responsibilities to account managers, he or she is ultimately responsible for procuring and managing the repository

(6) Pi is a mathematical constant equal to approximately 3.1415926535897932.

(7) The ratio of the circumference of a circle to its diameter; a number having a value to eight decimal places of 3.14159265.

1.3 Why \( \pi \)

"The story of pi reflects the most seminal, the most serious and sometimes the silliest aspects of mathematics. A surprising amount of the most important mathematics and a significant number of the most important mathematicians have contributed to its unfolding – directly or otherwise.

Pi is one of the few concepts in mathematics whose mention evokes a response of recognition and interest in those not concerned professionally with the subject. It has been a part of human culture and the educated imagination for more than twenty five hundred years.

The computation of Pi is virtually the only topic from the most ancient stratum of mathematics that is still of serious interest to modern mathematical research. And
to pursue this topic as it developed throughout the millennia is to follow a thread through the history of mathematics that winds through geometry, analysis and special functions, numerical analysis, algebra and number theory. It offers a subject which provides mathematicians with examples of many current mathematical techniques as well as a palpable sense of their historical development.

1.4 Precomputer history of $\pi$

That the ratio of circumference to diameter is the same (and roughly equal to 3) for all circles has been accepted as "fact" for centuries; at least 4000 years, as far as I can determine. (But knowing why this is true, as well as knowing the exact value of this ratio, is another story.) The "easy" history of $\pi$ concerns the ongoing story of our attempts to improve upon our estimates of $\pi$. This page offers a brief survey of a few of the more famous early approximations to $\pi$.

The value of given in the Rhynd Papyrus (c. 2000 BC) is $\frac{16\sqrt{2}}{9} = 3.160493827\ldots$ Various Babylonian and Egyptian writings suggest that each of the values were used (in different circumstances, of course). The Bible (c. 950 BC, 1 Kings 7:23) and the Talmud both (implicitly) give the value simply as 3.

Archimedes of Syracuse (240 BC), using a 96-sided polygon and his method of exhaustion, showed that

and so his error was no more than $\frac{1}{17} = 0.01408450704\ldots$

The important feature of Archimedes' accomplishment is not that he was able to give such an accurate estimate, but rather that his methods could be used to obtain any number of digits of $\pi$. In fact, Archimedes' method of exhaustion would prove to be the basis for nearly all such calculations for over 1800 years.

Over 700 years later, Tsu Chung-Chih (c. 480) improved upon Archimedes' esti-
mate by giving the familiar value
\[
\frac{355}{113} = 3.1415929203 \ldots
\]
which agrees with the actual value of \( \pi \) to 6 places.

Many years later, Ludolph van Ceulen (c. 1610) gave an estimate that was accurate to 34 decimal places using Archimedes’ method (based on a \( 2^{62} \)-sided polygon). The digits were later used to adorn his tombstone.

The next era in the history of the extended calculation of \( \pi \) was ushered in by James Gregory (c. 1671), who provided us with the series

Using Gregory’s series in conjunction with the identity

John Machin (c. 1706) calculated 100 decimal digits of \( \pi \). Methods similar to Machin’s would remain in vogue for over 200 years.

William Shanks (c. 1807) churned out the first 707 digits of \( \pi \). This feat took Shanks over 15 years – in other words, he averaged only about one decimal digit per week! Sadly, only 527 of Shanks’ digits were correct. In fact, Shanks published his calculations 3 times, each time correcting errors in the previously published digits, and each time new errors crept in. As it happened, his first set of values proved to be the most accurate.

In 1844, Johann Dase (a.k.a., Zacharias Dahse), a calculating prodigy (or "idiot savant") hired by the Hamburg Academy of Sciences on Gauss’s recommendation, computed \( \pi \) to 200 decimal places in less than two months.

In the era of the desktop calculator (and the early calculators truly required an entire desktop!), D. F. Ferguson (c. 1947) raised the total to 808 (accurate) decimal digits. In fact, it was Ferguson who discovered the errors in Shanks’ calculations.
Chapter 2

Methods Of Computation  \( \pi \)

2.1 Buffon’s Method

Buffon’s methods refers to a simple Monte Carlo method for the estimation of the value of \( \pi \), \( 3.14159265... \). The idea is very simple. Suppose you have a tabletop with a number of parallel lines drawn on it, which are equally spaced (say the spacing is 1 inch, for example). Suppose you also have a pin or needle, which is also an inch long. If you drop the needle on the table, you will find that one of two things happens: (1) The needle crosses or touches one of the lines, or (2) the needle crosses no lines. The idea now is to keep dropping this needle over and over on the table, and to record the statistics. Namely, we want to keep track of both the total number of times that the needle is randomly dropped on the table (call this \( N \)), and the number of times that it crosses a line (call this \( C \)). If you keep dropping the needle, eventually you will find that the number \( 2N/C \) approaches the value of \( \pi \! \)

Why does this work? It is not hard to show, with a little bit of calculus, that the probability on any given drop of the needle that it should cross a line is given by \( 2/\pi \). After many trials, the value of \( C/N \), the number of crossing needles divided by the total number of needles, will approach the value of the probability.
2.1.1 Monte Carlo Method

Introduction

The Monte Carlo method provides approximate solutions to a variety of mathematical problems by performing statistical sampling experiments on a computer. The method applies to problems with no probabilistic content as well as to those with inherent probabilistic structure. Among all numerical methods that rely on N-point evaluations in M-dimensional space to produce an approximate solution, the Monte Carlo method has absolute error of estimate that decreases as N superscript \(-1/2\) whereas, in the absence of exploitable special structure all others have errors that decrease as N superscript \(-1/M\) at best.

History of Monte Carlo method

The method is called after the city in the Monaco principality, because of a roulette, a simple random number generator. The name and the systematic development of Monte Carlo methods dates from about 1944. There are however a number of isolated and undeveloped instances on much earlier occasions.

For example, in the second half of the nineteenth century a number of people performed experiments, in which they threw a needle in a haphazard manner onto a board ruled with parallel straight lines and inferred the value of \(\pi = 3.14\) from observations of the number of intersections between needle and lines. An account of this playful diversion (indulged in by certain Captain Fox, amongst others, whilst recovering from wounds incurred in the American Civil War) occurs in a paper Hall (A. HALL 1873. ”On an experimental determination of \(\pi\”). The author of this WEB page has developed the software for simulating of this experiment. Try JAVA implementation of Buffon’s needle experiment for the determination of \(\pi\).

In 1899 Lord Rayleigh showed that a one-dimensional random walk without ab-
sorbing barriers could provide an approximate solution to a parabolic differential
equation.

In 1931 Kolmogorov showed the relationship between Markov stochastic processes
and certain integro-differential equations.

In early part of the twentieth century, British statistical schools indulged in a fair
amount of unsophisticated Monte Carlo work. Most of this seems to have been of
didactic character and rarely used for research or discovery. Only on a few rare oc-
casions was the emphasis on original discovery rather than comforting verification.

In 1908 Student (W.S. Gosset) used experimental sampling to help him towards his
discovery of the distribution of the correlation coefficient. In the same year Student
also used sampling to bolster his faith in his so-called t-distribution, which he had
derived by a somewhat shaky and incomplete theoretical analysis.

The real use of Monte Carlo methods as a research tool stems from work on the atomic
bomb during the second world war. This work involved a direct simulation of the
probabilistic problems concerned with random neutron diffusion in fissile material;
but even at an early stage of these investigations, von Neumann and Ulam refined
this particular "Russian roulette" and "splitting" methods. However, the systematic
development of these ideas had to await the work of Harris and Herman Kahn in
1948. About 1948 Fermi, Metropolis, and Ulam obtained Monte Carlo estimates for
the eigenvalues of Schrodinger equation.

In about 1970, the newly developing theory of computational complexity began to pro-
vide a more precise and persuasive rationale for employing the Monte Carlo method.
The theory identified a class of problems for which the time to evaluate the exact
solution to a problem within the class grows, at least, exponentially with M. The
question to be resolved was whether or not the Monte Carlo method could estimate
the solution to a problem in this intractable class to within a specified statistical
accuracy in time bounded above by a polynomial in M. Numerous examples now
support this contention. Karp (1985) shows this property for estimating reliability in
a planar multiterminal network with randomly failing edges. Dyer (1989) establish
it for estimating the volume of a convex body in M-dimensional Euclidean space.
Broder (1986) and Jerrum and Sinclair (1988) establish the property for estimating the permanent of a matrix or, equivalently, the number of perfect matchings in a bipartite graph.

How to use Applet

2.1.2 Buffon’s Needle Methods

Buffon’s needle experiment is a very old and famous random experiment, named after Compte De Buffon. The experiment consists of dropping a needle on a hardwood floor. The main event of interest is that the needle crosses a crack between floorboards. Strangely enough, the probability of this event leads to a statistical estimate...
of the number pi!

**Assumptions**

Our first step is to define the experiment mathematically. Again we idealize the physical objects by assuming that the floorboards are uniform and that each has width 1. We will also assume that the needle has length $L \geq 1$ so that the needle cannot cross more than one crack. Finally, we assume that the cracks between the floorboards and the needle are line segments.

The Monte Carlo method provides approximate solutions to a variety of mathematical problems by performing statistical sampling experiments on a computer. The method applies to problems with no probabilistic content as well as to those with inherent probabilistic structure. Among all numerical methods that rely on $N$-point evaluations in $M$-dimensional space to produce an approximate solution, the Monte Carlo method has absolute error of estimate that decreases as $N^{-1/2}$ whereas, in the absence of exploitable special structure all others have errors that decrease as $N^{-1/M}$ at best.

The method is called after the city in the Monaco principality, because of a roulette, a simple random number generator. The name and the systematic development of Monte Carlo methods dates from about 1944.

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The real use of Monte Carlo methods as a research tool stems from work on the atomic bomb during the second world war. This work involved a direct simulation of the probabilistic problems concerned with random neutron diffusion in fissile material; but even at an early stage of these investigations, von Neumann and Ulam refined this particular "Russian roulette" and "splitting" methods. However, the systematic development of these ideas had to await the work of Harris and Herman Kahn in 1948. About 1948 Fermi, Metropolis, and Ulam obtained Monte Carlo estimates for the eigenvalues of Schrodinger equation.

INTRODUCTION

Buffon’s Needle is one of the oldest problems in the field of geometrical probability. It was first stated in 1777. It involves dropping a needle on a lined sheet of paper and determining the probability of the needle crossing one of the lines on the page. The remarkable result is that the probability is directly related to the value of pi. Suppose you have a tabletop with a number of parallel lines drawn on it, which are equally spaced (say the spacing is 1 inch, for example). Suppose you also have a pin or needle, which is also an inch long. If you drop the needle on the table, you will find that one of two things happens:

(1) The needle crosses or touches one of the lines
(2) the needle crosses no lines.

The idea now is to keep dropping this needle over and over on the table, and to record the statistics. Namely, we want to keep track of both the total number of times that the needle is randomly dropped on the table (call this tries), and the number of times that it crosses a line (call this hits). If you keep dropping the needle, eventually you will find that the number $2n/h$ approaches the value of pi!

Methods:
Let’s take the simple case first. In this case, the length of the needle is one unit and
the distance between the lines is also one unit. There are two variables, the angle at which the needle falls (theta) and the distance from the center of the needle to the closest line (D). Theta can vary from 0 to 180 degrees and is measured against a line parallel to the lines on the paper. The distance from the center to the closest line can never be more than half the distance between the lines. The graph below depicts this situation.

\[ D \leq \frac{1}{2} \sin(\theta) \]

The needle in the picture misses the line. The needle will hit the line if the closest distance to a line (D) is less than or equal to 1/2 times the sine of theta. That is, \( D \leq \frac{1}{2}\sin(\theta) \). How often will this occur?

\[ \text{The shaded portion is found with using the definite integral of } \frac{1}{2}\sin(\theta) \text{ evaluated from zero to } \pi. \]

The value of the entire rectangle is \( \frac{1}{2}\pi \) or \( \pi/2 \). So, the probability of a hit is \( \frac{1}{\pi/2} \) or \( 2/\pi \). That’s approximately .6366197.

To calculate pi from the needle drops, simply take the number of tries and multiply
it by two, then divide by the number of hits, or

\[ \frac{2 \text{(total tries)}}{\text{(number of hits)}} = \pi \text{(approximately)}. \]

There are three types of Needle method for calculate value of pi:

(1) Simple Method
(2) Frame Method
(3) Table Method

\subsection*{2.1.2.1 Simple Needle Method}

How to use this Applet....

The left hand side shows the tabletop with parallel lines, being covered with needles randomly. First select number of needles, Click on “Combo”, then after Click on ”Start” button to begin. Note that if you click on ”Start,” again, it will begin the simulation from scratch. The bar graph in the middle displays the value of the current estimate over a smaller range. When you begin, the estimated value will probably be outside of this range. The plot on the right shows the estimated value versus the log of the number of needles thrown, on a wider scale.
2.1.2.2 Needle Method using Frame

How to use this Applet...

First select number of needles, Click on "Combo" then after Click on "OK" button will open Frame. The left hand side shows the tabletop with parallel lines, being covered with needles randomly. Click on "Start" button to begin. Note that if you click on
“Start,” again, it will begin the simulation from scratch. The bar graph in the middle displays the value of the current estimate over a smaller range. When you begin, the estimated value will probably be outside of this range. The plot on the right shows the estimated value versus the log of the number of needles thrown, on a wider scale.
2.1.2.3 Needle Method using Table

How to use this Applet...

The left hand side shows the tabletop with parallel lines, being covered with needles randomly. Right hand side select number of needles and select number of times It means how many times wants to falls needles on table, then click on "Start" button
to begin, after click on ”Next” button and again click on ”Start” button and process continue. After click on ”Display” button it will show table with the value of pi in each iteration and also calculate total average value of pi. and if wants to repeat this process again then click on ”Refresh” button and continue. The bar graph in the middle displays the value of the current estimate over a smaller range. When you begin, the estimated value will probably be outside of this range. The plot on the right shows the estimated value versus the log of the number of needles thrown, on a wider scale.
2.2 Ramanujan Method

Srinivasa Ramanujan was one of India’s greatest mathematical geniuses. He made substantial contributions to the analytical theory of numbers and worked on elliptic functions, continued fractions, and infinite series.

Ramanujan was born in his grandmother’s house in Erode, a small village about 400 km southwest of Madras. When Ramanujan was a year old his mother took him to the town of Kumbakonam, about 160 km nearer Madras. His father worked in Kumbakonam as a clerk in a cloth merchant’s shop. In December 1889 he contracted smallpox.

When he was nearly five years old, Ramanujan entered the primary school in Kumbakonam although he would attend several different primary schools before entering the Town High School in Kumbakonam in January 1898. At the Town High School, Ramanujan was to do well in all his school subjects and showed himself an able all round scholar. In 1900 he began to work on his own on mathematics summing geometric and arithmetic series.

Ramanujan was shown how to solve cubic equations in 1902 and he went on to find his own method to solve the quartic. The following year, not knowing that the quintic could not be solved by radicals, he tried (and of course failed) to solve the quintic.

It was in the Town High School that Ramanujan came across a mathematics book by G S Carr called Synopsis of elementary results in pure mathematics. This book, with its very concise style, allowed Ramanujan to teach himself mathematics, but the style of the book was to have a rather unfortunate effect on the way Ramanujan was later to write down mathematics since it provided the only model that he had of written mathematical arguments. The book contained theorems, formulas and short proofs. It also contained an index to papers on pure mathematics which had been published in the European Journals of Learned Societies during the first half of the 19th century. The book, published in 1856, was of course well out of date by the time Ramanujan used it.

By 1904 Ramanujan had begun to undertake deep research. He investigated the series
(1/n) and calculated Euler’s constant to 15 decimal places. He began to study the Bernoulli numbers, although this was entirely his own independent discovery.

2.2.1 Ramanujan’s Circle Method

Introduction This method of computing is given by great mathematician Ramanujan. Ramanujan give his own formula to calculate the value of using circle. The method is given below:

Let PQR be a circle with center O, of which a diameter is PR. Bisect PO at H and let T is the point of trisection OR nearer R. Draw TQ perpendicular to PR and place the chord RS=TQ.

Join PS, and draw OM and TN parellel to RS. Place a chord PK=PM, and draw the tangent Pl =MN. Join RL,RK and KL. Cut off RC=RH. Draw CD parrel to KL,meeting RD at D.

Then the square on RD will be equal to circle PQR approximately.

For $RS^2 = 5/36d^2$
where $d$ is diameter of circle.
Therefore $PS^2 = 31/36d$
But PL and PK are equal to MN and PM respectively.
Therefore $PK^2 = 31/144d^2$, and $PL^2 = 31/324d^2$.
Hence $RK^2 = PR^2 - PK^2 = 113/144d^2$,
and $RL^2 = PR^2 + PL^2 = 355/324$.
But RK/RL = RC/RD=3/2 sqrt(113/355) and RC=3/4d.

Note:- If the area of the circle be 1450,000 square miles, then RD is greater than true length by an inch.

**how to use this Applet**

Use of this Applet is very simple. You have to only select the diameter of the circle
through combo box and in this Applet all the lines will be drawn automatically and the value of $\pi$ will be calculated. Actually in this method all the thing is based on the co-ordinate geometry. We have to calculate the co-ordinate when drawing the line.

### 2.2.2 Ramanujan’s Digit Method

The applet calculates Pi using Ramanujan’s series

$$\frac{1}{\pi} = \frac{\sqrt{8}}{9801} \sum_{n=1}^{\infty} \frac{(4n)!(1103+26390n)}{(n!)^4 396^{4n}}$$

There are three main calculations in this applet.

1. Taylor series for the square root of 8.
2. Ramanujans series for $\frac{1}{\pi}$.
3. Newton Rapson for the reciprocal.

Multiple precision numbers are implemented arrays of integers, each representing a radix -10000 digit.

**How to use this Applet** First of all you have to click on Click here button then it will display digit and continue this process until digit will reach up to 10000.
\[ \pi = 3.14159265358979323846264338327950288419716939937510582 \]

\[ 4459230781640628620899862803482534211706798214808651328230 \]

\[ 84460955058223172535940812848111745028410270193852110555964 \]

\[ 8954930381964428810975665933446128475648233786783165271201 \]

\[ 4856692346034861045432664821339360726024914127372458700660 \]

\[ 17488152092096282925409171536436786295903600113305305488204 \]

\[ 4146951941511609433057270365795959195309218611738193261179310 \]

\[ 4807446237996274956735188575272489122793818301194912983367 \]

\[ 6566430860213949463952247371907021798609437027705392171762 \]

\[ 2384674818467669406132000568127145263560827785771342757789 \]

\[ 3717872146844090122495343014654958537106079227968925892354 \]

\[ 112129021960864034418159813629774771309960518707211349999999 \]

\[ 80499510597317328160963185950244659455346908302642522308253 \]

\[ 3526193118817101000313783875288658875332083814206171776691473 \]

\[ 2534904287554687311595628638823537875937519577818577805321 \]

\[ 66130019278766111959092164201989380952572010654868632788659 \]

\[ 8182796823030195203530185296899577362259941389124972177528 \]

\[ 5155748572424541506959508295331168617278558890750983817546 \]

\[ 9319255060400927701671139009848882401285836176 \]
2.3 Archimedes’s Method

Archimedes (approximately 285—212 B.C.) was the most famous ancient Greek mathematician and inventor. He invented the Screw of Archimedes, a device to lift water, and played a major role in the defense of Syracuse against a Roman Siege, inventing many war machines that were so effective that they long delayed the final sacking of the city.

Archimedes’s proposed that the value of $\pi$ will be between perimeter of polygon drawn Incircle and Outcircle of unit length. Means perimeter of incircle $< \pi <$ perimeter of outcircle.
<table>
<thead>
<tr>
<th></th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perimeter of Incircle</td>
<td>3.199999999999993</td>
</tr>
<tr>
<td>Perimeter of Outcircle</td>
<td>4.199999999999999</td>
</tr>
<tr>
<td>Difference</td>
<td>1.0</td>
</tr>
</tbody>
</table>

**Diagram:** A geometric figure with a circle inscribed in a polygon and another circle circumscribed around it. The difference in their perimeters is highlighted.
In the above applet, we are drawing incircle and outcircle of different no of edges and calculating its perimeter. As you increase the no of edges it’s perimeter is going to closer to the circle and at a point both incircle and outcircle perimeter is same and at this point you will get most approximate value of $\pi$.

### 2.4 Circumference Method

**Introduction** A circle’s circumference or perimeter is the distance around the circle. This is shown in red in the figure below. Starting at point A, going all around the circle, and ending on the same point A represents the circumference of the circle. The diameter is the longest line segment that you can have inside a circle. These line segments go through the center of the circle and have their endpoints on the circle. The previous picture shows three examples of diameters in green, yellow, and brown. Now that you know what circumference and diameter are, here are a few questions for you.

1. Think of many circles of different sizes. Do you think there is a relationship between the circumference and the diameter?
2. If so, what might the relationship be?
3. Looking at the figure above, how many times larger do you think the circumference is than its diameter? Try to estimate a number.

Do the measurements. At the bottom of this page you will find an applet that you will use to make many circles and measure their circumference and diameter. You should do about 10 circles with diameters that vary between 10 and 150 units. Make sure to try a circle with a diameter of 70 units. When measuring the circumference, make sure that the starting point (where the small line in the circle is) goes around a complete cycle until it ends as close to the center and as close to vertical as possible. You need to record both measured quantities in a table. You can do this table in three different ways:

1. On a piece of paper,
2. Using a spreadsheet program in your computer, like excel, or
3. Using the table provided with this lesson.

If you choose to use the table provided, be aware that if you exit from that page
you will loose all the results. You can certainly fill it in and print it, but as soon as
you exit from it, the data is gone. You need to leave the table page open as you go
to the next page of this lesson because you will need it there.
Now go ahead and click on the symbol in the applet to read the instructions on how
to use it, measure both dimensions for 10 circles, and fill out the first two columns of
the table. When done, go on to the next page to analyze the data.
In this method we have to select a diameter of the circle and there is a dot on the
circle. We will rotate the circle then dot will also rotate on along the circle. We
will rotate the circle until dot is reached up to start point and at this it gives the
perimeter for selected diameter.

Through above method, we can calculate at most ten values of \( \pi \) for different di-
ameters of the circle and thus we get the better approximation of \( \pi \) the by calculating
the average value of \( \pi \).

You should have noticed that the ratios for all the circles are very close. The values
that you should have gotten should be very close to 3.1. The measurements that you
did were not that precise. If they had been, then all of the ratios would be extremely
close to each other.
What that tells us is that there is a fundamental constant that works with every sin-
gle circle. The name of this constant is pi and its value is close to 3.1415926535897932...
The Greek letter is used to represent this important constant. One more thing that
you may want to do with your data is get the mean or average of all the ratios (or
approximations of ) that you measured to see how close you are to the value of in the
previous paragraph.
How to use this Applet

Concept of calculating the value of \( \pi \) based on formula that circumference = \( 2 \times r \), where \( r \) is the radius of the circle. Or in other way we can say that circumference = diameter \( \times \pi \). It means if we know diameter and perimeter, then we can find the value of \( \pi \).

Here in this Applet, we shall select the diameter, then the circle of the selected diameter will be drawn. Then click on start button and scroll the horizontal scroll bar, until red dot is reached again on the horizontal and some value appears in diameter and circumference text box. At this time vertical scroll bar and start button is inactive. Now click on the next button to see the value of \( \pi \). In this way we can see the ten different values of \( \pi \), during ten iteration and its average value will be calculated each time. In the mean time if we want to start again then we can click on the refresh button.
CURRENT DIAMETER -> 0.0
CURRENT CIRCUMFERENCE -> 0.0
2.5 Rob’s Computer Method

Introduction

The above applet implements a few methods for computing pi. You can pick your method from the combo box. All of them are series or iterative methods, so click on reset to reset to first order estimate, single step to step forward one more step, or you can enter a number in the multi step box and click the button to jump forward that many steps all at once.

For all these methods only basic operations (arithmetic and square root) are used. You might argue that square root is not a basic operation, but I think I could write a recursive square root function for you using only arithmetic operations.

The Trapezoidal Integral method uses the trapezoidal rule to compute the area of a quarter circle of radius 1, which is pi/4. This method converges slowly, but keeps getting closer.

The Binomial Integral method expands the integrand for the area of a quarter circle, written in cartesian coordinates, as a series using the binomial expansion. The integral of each polynomial term is easily computed analytically, yielding a series for pi. This converges rapidly, but it has problems after a few iterations because there are factorials in the series which blow up quickly and start to overflow the "long" data structures.

The Newton-Raphson method expands cos(x) as the well-known series, and uses Newton-Raphson to iteratively search for a zero, each time adding another term to the series. The zero of the function is pi/2. This method has similar convergence,
and suffers from a similar problem to the Binomial Integral method.

The Arctan Series method expands $\arctan(x)$ as a Taylor series about $x=0$, and then evaluates it at $x=1$, producing the simple series $1-1/3+1/5-1/7...$ which converges to $\pi/4$. Convergence is very slow, worse than the Trapezoidal Integral method. This series does not suffer from the overflow problem because there are no factorials or other rapidly growing numbers. This is the only method so far which does not use the square root function. This method was Ron Walker’s idea.

The Iterative Cosine Series method was written by Alan Asbeck. He uses a sequence of approximations to $\pi/2$ by $guess[i]=guess[i-1]+\cos(guess[i-1])$ where cosine is computed using a Maclaurin series about $\pi/2$. He uses 20 terms in the Maclaurin series to compute the cosine, which is why you will notice that the Java App says each ”single step” counts for 20 steps.

**How to use this Applet**

First of all click on combo box and select method which method wants for calculate value of Pi., after selected method click on Single Step button if you want calculate value in each step then you have to click on Single Step button otherwise if you want calculate value of pi in multiple step then you have to click on Multi Step button, and continue click on button until no iteration finish. Every step displays the value of pi and also display estimate error. If you want to stop processing then you have to click on RESET button, then process will be go stop.
2.6 Area’s Method

**Introduction** This method is also known as Integration through Decreasing Function. If the curve slopes downward, then the altitude of each inscribed rectangle is the length of its right edge, and we have areas as follows:

First rectangle \( f(x_1) \cdot \Delta \)

Second rectangle \( f(x_2) \cdot \Delta \)

Third rectangle \( f(x_3) \cdot \Delta \)

\[ \vdots \]

Nth and last rectangle \( f(x_n) \cdot \Delta \)
More generally, the curve may rise and fall between \( x = a \) and \( x = b \). But there is a number \( c_1 \) between \( a \) and \( x_1 \) inclusive, such that the first inscribed rectangle has area \( f(c_1) \cdot \Delta x \) and number a number \( c_2 \) in the second closed subinterval such that the area of the second inscribed rectangle is \( f(c_2) \cdot \Delta x \); and so on.

The sum of areas of these inscribed rectangle is

\[
S_n = f(c_1) \cdot \Delta x + f(c_2) \cdot \Delta x + f(c_3) \cdot \Delta x + \ldots + f(c_n) \cdot \Delta x.
\]

It can be written as

\[
S_n = \sum_{k=1}^{n} f(c_k) \cdot \Delta x
\]

Here this method is named as calculatePi() method.

The calculatePi() method loops forever in an attempt to find and capture the elusive pi, the ratio of the circumference of a circle to its diameter. To calculate pi, the calculatePi() method tries to determine the area of a circle that has a radius of one. Because the circle has a radius of one, the circle’s area is pi itself.

To determine the area of the circle with radius one, the calculatePi() method works to find the area of one fourth of the circle, then multiplies that area by four to get pi. Here’s a diagram of a circle with the area that calculatePi() focuses on shown in blue:
To find the area of the portion of the circle shown in blue in the diagram above, the calculatePi() method slices the area up into progressively smaller rectangular segments, as shown in the diagram below:

Because calculating the area of a rectangle is a piece of cake (or in this case, a slice of pi), the calculatePi() method is able to approximate pi by calculating and summing the areas of the rectangles, then multiplying the result by four. As you can see from the above diagram, this approach to calculating pi will always yield an ap-
proximation of pi that is too large. As the rectangles get thinner and more numerous, the approximation will get closer and closer to the real pi.

The calculatePi() method works by making repeated passes at calculating the area of the quarter circle, with each subsequent pass using a smaller slice width. Each iteration of calculatePi()’s for loop represents one attempt to calculate the area of the quarter circle. For any particular iteration of the for loop, the slicewidth variable gives the x (horizontal) width of the slice, which remains constant during the entire iteration of the for loop. At the end of each for loop iteration, the slice width is halved.

The value of the pi variable keeps a running record of the current approximation of pi. Each pass of calculatePi()’s for loop starts by initializing the x local variable to 0.0, then incrementing x by slicewidth until x reaches the end of the circle’s radius at 1.0.

The while loop that is contained inside the for loop iterates once for every two slice widths. In effect, each iteration of the while loop takes a large rectangle from the previous iteration of the for loop, discards half of the large rectangle’s area, and calculates a new value for the discarded portion. Because the slice width is halved at the end of each iteration of the for loop, the width of the rectangles calculated by the previous iteration of the for loop is always twice as wide as the slice width of the current iteration. Here is a diagram showing the steps the calculatePi() method takes to divide two rectangles into four:
As you know that $area = \pi r^2$. So if we know the area of the circle and its radius, we can calculate the value of pi. In this method, we take a quarter of the circle and divide it into different equal parts and we can calculate area by integration of all the parts. So if we are getting 1/4 th area then multiplying by $4/r^2$, we can get the value of $\pi$.

In this method, if division of the circle is more, approximate area is closer and then $\pi$’s value is more approximate. In this applet, we are dividing the area using different method. In this Applet division of circle is in multiple of two for calculating conven-
tion. The Applet is divided in 1.2.4..........................256 parts. All the methods are following. 1. Mid Point Method
2. Left End Point Method
3. Right End point Method
4. Simpson Method
5. Trapezoidal Method

Now we shall discuss all above methods in detail.
In this method we divide the circle in the no of parts selected and height of the rectangle is considered as the mid point of the both end of the rectangle. This method gives better approximation than other methods.
In Left end point method the height of the rectangle is calculated at the left point of the rectangle. It also gives less better approximation than Mid point method but gives better approximation than Right end point method.
In Right end point method, the height of the rectangle is calculated as the right end point of the circle. This method gives less approximate method.
This method is given by Simpson. In this method we have to draw arc instead of rectangle, which require at least three points. So after dividing the circle we will draw the arc and calculate the area using Simpson method.

Simpson’s Rule for calculating area is given as:-
\[ \int_a^b f(x) \, dx = \frac{h}{3} \left[ y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \ldots + 2y_{n-2} + 4y_{n-1} + y_n \right] \]

where \( h = (a-b)/n \).

In this Trapezoidal method, we join the top of the rectangle by straight lines instead of curves and calculate the area using Trapezoidal method. Trapezoidal
method is given below.

\[ \int_{a}^{b} f(x) \, dx = \frac{h}{2} \left[ y_0 + 2y_1 + 2y_2 + 2y_3 + 2y_4 + \ldots + 2y_{n-1} + y_n \right] \]

where \( h = \frac{(b-a)}{2n} \).

**How to use this Applet** In this method, first of all we shall select one of the five radio buttons, which identify the method through which we have to calculate the value of \( \pi \). The selected radio button will be red and rest of the buttons will be green. If at any moment of time we select different radio button, then selected radio button will be red and others will be green. Now we will select the no of parts in which we have to divide the circle. It will be in the form of 1.2.4.256 according to programming convention. Means it is in multiple of two and then its area is calculated internally and gives the value of \( \pi \) in the text box. The value of \( \pi \) will differ in different methods for the number of same parts.